

Presentation of a Pi number using a computer

Rešić Sead, Čajić Elvir, Osmić Amila, Omerović Maid

Abstract. The story of the number π is known from the very beginning of mathematics. His name less familiar Ludof's number, as named by Ludolph van Ceulen who first calculated the number of π accurate to 35 decimal places.

Mentioned number has an infinite decimal notation aperiodic and therefore is practiced by many mathematicans to determine the decimal number where more will be discussed in this paper.

The ratio of the volume and the radius of the circle can be expressed in many ways through the various formulas used in geometry ,calculus,probability and physics where there are algorithms for computation. Using a computer with the algorithms can reach several milion decimal number. In this work number π determines in computers softwer Mathematica.

Keywords: Number pi, method of determining the number pi, algorithm, computer software.

1. Introduction

Pi or π is a mathematical constant, widely used today in mathematics and physics. It is defined as the ratio of the volume and circle diameter. Pi is also known as the Archimedes constant (it is not necessary to mix it with Archimedes number) or Ludo's number.¹ In practice, it is written with a small Greek letter π and in the Bosnian language it is proper to write and write. The 64 decimal numerical value pi is rounded to places is: 3.14159 23846 83279 69399 Π ≈ 26535 89793 26433 50288 41971 37510 58209 74944 5923 The history of the knowledge of the number π runs parallel with the development of the mathematics itself. Some authors share the progress in this field in three periods: antique - in which it is calculated geometrically, classically - which was calculated by using more mathematics in Europe around the 17th century and the third period - digital computing time on computers.² The "pi" is derived from the Greek word "perimeter" (TEP/µETPOS), which means the length of the curve that limits a plane. This symbol was first introduced by mathematician William Jones in 1706, although the mark was fully accepted and became popular only after it was adopted by Leonard Euler.³

2. Liu Hui Pi algorithm

Later, Liu Hui invented Liu Hui π algorithm and achieved $\pi = 3.1416$ using a polygon of only 96 pages, taking advantage of the fact that the difference in surfaces of successive polygons forms a geometric series with factor. About 480, Chinese mathematician Zu Chongzhi gave an approximation of $\pi = 355/113$ and showed that $3.1415926 < \pi < 3.1415927$. It was obtained using Liu Hui's π algorithm for a 12288-page polygon. This will prove to be the most accurate calculation of the number over the next 900 years.⁴

3. Classic period

By the second millennium, π was known to less than 10 decimal places. The next major advance took place with the emergence of more mathematics, and in particular the discovery of infinite sequences. These arrays theoretically allow you to calculate π to any desired value by adding the required number of members. ⁵About the year 1400, Madhava of Sangamagram discovered the first known series of this kind.Madhava managed to calculate to the next accuracy to 11 decimal places

$\pi = 3.14159265359.$

The record was invaded in 1424 by the Persian astronomer Jamshīd al-Kāshī, who calculated 16 decimals of the number π . The first major advance in calculating the number π after Archimede was made by German mathematician Ludolph van Ceulen (1540-1610), who used the Archimedes method and a polygon with 60 \cdot 2029 pages to calculate 35 decimals of the number π . He was so proud of the calculation, which required most of his life, to give the digits to his tombstone. At about the same time, methods of more mathematics and the determination of infinite series began to spin over Europe. The first of the known types is the Vinte formula discovered by Franzois Vinte in 1593.

Formulas of this kind, now known as the machine formula, have been used to set up several successful records and others remembered as the best-known computing methods at the time of the computer.

The number of p in the era of the computer. Symbolic mathematical programs such as Maple or Mathematica can calculate 10,000 decimals of number p at the moment, and another 20,000-1,000,000 decimals at night (the ranking depends on the platform that the computer owns). Brothers Chudnovsky of New York calculated 2 trillion decimals of number p on a homebrew computer. The current record for the number of decimals calculated is Yasumas Canada and Daisuke Takahashi from the University of Tokyo. This is about 206 trillion decimals of p (206,158,430,000 decimal places). This is how your account

 $^{^1}$ Archimedes' constant $\pi,\ 2007\text{-}11\text{-}04$

² Groleau, Rick. Infinite Secrets: Approximating Pi. NOVA, 2007-11-04 2.Eymard, Pierre; Jean-Pierre Lafon (02 2004). "26", *The Number* π , Stephen S. Wilson (translator) (na English), str. 53, American Mathematical Society. 2007-11-04. ISBN 0821832468

³ A. KLOBUCAR, Priča o broju π, Matematičko fizički list, 189 (1997),1-5.

⁴ Charles Hutton (1811). *Mathematical Tables; Containing the Common, Hyperbolic, and Logistic Logarithms...*, str. p.13, London: Rivington.

⁵ Beckmann, Petr (1976). A History of π, St. Martin's Griffin. ISBN 0-312-38185-9



was presented by Yasumas Canada, from the University of Tokyo. There are three essential methods for calculating a number of values p with many decimals:

1. One of the oldest in use is with the help of a growing string tg-1 (x) = x-x3/3 + x5/5 ...together with a formula such as p = tg-1(1/5)-4 * tg-1(1/239). That account gives about 1.4 decimals per member.

2. The second formula in use comes from the Aritmetic-Geometry through the account. An excellent overview of these formulas is given in the book "p and AGM". Their advantage is that they double the number of decimals by iteration. For example, in order to achieve 1 000 000 decimals, about 20 iterations are quite sufficient. The disadvantage is because we need a FFT type of doubling to get a reasonable speed, and this is not so easy for the programs.

3. The third comes from the theory of complex multiplication of elliptic curves, and was discovered by S. Ramanujan. This gives a great number of beautiful formulas, but the most useful was missing Ramanujan and was discovered by Chudnovsky's. It reads like this (slightly modified for easy programming):

Set k_1 = 545140134; k_2 = 13591409; k_3 = 640320; k_4 = 100100025; k_5 = 327843840; k_6 = 53360;

Then $pi = (k_6 \operatorname{sqrt}(k_3))/(S)$, where

 $S = sum_{(n=0)} co(-1) n ((6n)! (K_2 + nk_1)) / (n!^3 (3n)! (8k_4k_5) n)$. The great advantage of this formula is in the following

1) It converges linearly, but very fast (over 14 decimal places per member)

2) The way it's written, all the operations for computing S are written very easy to program. For this reason, the constant $8k_4k_5$ that appears as the denominator is written in this way instead of 262537412640768000. Thus, Chudnovsky's managed to calculate several trillion decimals.

3) One interesting new method is suggested by David Bailey, Peter Borwein and Simon Plouffe. It can calculate n hexadecimal decimal p efficiently without the previous (n-1) decimal. The method is based on the formula: $pi = sum_{(i = 0)} co(1 16 i)((4 8i + 1) - (2 8i + 4) - (1 8i + 5) - (1 8i + 6))$

4) for O (n) time and O (log N) space. Here's what the 160 C-program looks like, written by Dick T. Winter in CWI, which counts up to 800 decimal places. inta = 10000, b, c = 2800, d, e, f [2801], g;

5) main () {for (; b-c;) f [b++] = a / 5; for (; d = 0, g = c * 2; c = 14,

6) printf ("% 4d", e + d/a), e = d% a) for (b = c; d + = f[b] * a, f[b] = d% g -, - b; d * = b)}

Through the Internet (e-mail) Yasumasa Canada commenting on his previous record of some 50 trillion decimals, he wrote:"Dear people, The moment has come when we can publish a new world record related to p. It took longer than our expectation. Nearly two years have passed since we got a new world record of 6.4 trillion decimals. Now, we have an eight times higher record of 6.4 trillion. Yasumasa Canada, Computer Center, University of Tokyo. "On September 20, 1999 Yasumasa Canada released a new record. This time the so-called. Canada Laboratory (Canada Laboratory) announced that they have reached a known number of 206,158,430,000 decimal places. And this time they made their account using two different methods. More precisely, they calculated

 $3 * 236 = 206\ 158\ 430\ 208$ decimal places, from which the account matched 206\ 158\ 430\ 16\ days.

Here's another interesting thing about p. 707 decimals printed on a frieze of a round room, dedicated to number p, at the Palace of Discoveries in Paris. The palace was built for the great World Exposition in 1900, just as the Eiffel Tower was erected.⁶

4.Number Pi in Mathematica software and algorithms for calculating the number of pi in Mathematica software 4.1. An analysis, tools and methods for calculating the number of pi.

In this paper, as we have already shown, there are many ways to calculate the decimals of the pi number. This part of our work is our contribution using Mathematica software. In addition to the basic commands for calculating boring pi, the algorithms we have programmed in this paper were used. The algorithm is a set of steps and procedures that are clearly defined and which, as a result, give us a final solution to our problem. So Mathematica software is the tool used to calculate the number of pi.Softver Mathematica can us by typing the command N [Pi, 8] eight decimals.

So the first technique for calculating the number of pi to eight decimal places is achieved by the command:

N[Pi, 8] = 3.1415926. So the Mathematica user interface consists of two commands in the In window: we enter the formula and terms that we can define as software users. In the upper corner of the software there is the Run packeg command, and if we have well defined the parameter the result of our command is Out and there is in this case, 3.1415926 eight decimals of the Pi number. The calculation calculator in this case is N, which means that N is the operator that calls to calculate the decimals of the Pi number. In the case that the command N [Pi] caught the mathematical interface, it will give us as a result of 6 digits of the decimal number of the Pi number. It gives us the right to have the suspicion that the Pi function in Mathematica software has already been created using an algorithm. Thus, after entering N [Pi, 100], we get one hundred decimal places in the Mathematica software. By studying the software and by deeper analysis, the software developer and software owner put in an algorithm used in embedding Pi in Mathematica is: Brent-Salamin formula, also called Gauss-Salamin formula or Salamine formula, is a formula that uses an arithmetic-geometric mean to calculate the value of pi. It has square convergences.

Let it go

$$\frac{1}{2}(a_n+b_n)$$

a

⁶ D. H. BAILEY, J. M. BORWEIN, P. B. BORWEIN, Ramanujan, Modular Equations, and Approximation to Pi or How to compute One Billion Digits of Pi



and define the initial conditions to be, $a_0 = 1, b_0 = 1/\sqrt{2}$. then repeating it and giving an arithmetic-geometric mean M(a, b), and π gives.

$$\frac{4\left[M\left(1,2^{-1/2}\right)\right]}{1-\sum_{j=1}^{\infty}2^{j+1}d}$$
$$\frac{4\left[M\left(1,2^{-1/2}\right)\right]}{1-\sum_{j=1}^{\infty}2^{j+1}c}$$

By deeper analysis and having the ability to reason, typing in a series of commands after several months, I came to the conclusion that in the Mathematica software, bor's Pi can be calculated to only 10,000 decimals. And this typing command N [Pi, 10000] is the result:





By typing the command N [Pi, 10000] the time for which this Mathematica software was calculated was not more than 1.5 seconds. Considering that we own an Ouad-Core 3.00 Ghz, 4Gb ram, 2gb GPU card and 1TB terabytes hard drive for this computer is relatively fast. Further analysis found that the inclusion command N [Pi, 10001] as a result of the Mathematica software package results in an error. Which means it is impossible to execute the command. However, by opening notebook windows and entering the program code and algorithm:

def pi_chudnovsky(one=1000000):

Calculate pi using Chudnovsky's series

This calculates it in fixed point, using the value for one passed in

```
k = 1
a k = one
a sum = one
b sum = 0
C = 640320
C3 OVER 24 = C^{**3} // 24
while 1:
a k = -(6*k-5)*(2*k-1)*(6*k-1)
a k //= k*k*k*C3 OVER 24
a sum += a k
b sum += k * a k
k += 1
if a k == 0:
break
total = 13591409*a sum + 545140134*b sum
pi = (426880*sqrt(10005*one, one)*one) // total
return pi
II dio
def sqrt(n, one):
```



,,,,,,

Return the square root of n as a fixed point number with the one passed in. It uses a second order Newton-Raphson convergence. This doubles the number of significant figures on each iteration.

Use floating point arithmetic to make an initial guess

The Mathematica software package is now able to calculate the number of pi to 10,000,000 decimals. The problem is still in time and memory size as well as the speed of calculating the number Pi, which can be done with the help of updating the computer equipment. The following table gives the time for which the number Pi is calculated as well as the number of decimals

So we know the well-known equation and Chudnovsky algorithm in mathematical form, we have programmed into Mathematica software and he is now able to calculate the number Pi to 10 million decimals. The result can be even bigger and go into a trillion combinations, however, a more modern machine that can to withstand so much load. It is also possible to get at the speed by typing in different algorithms. The algorithm in this case is only described by the mathematical problem, since the convergence criterion of this order is chosen, therefore, it is closely related to the number of decimals this program package can calculate. converges to more decimals, and therefore the number of decimals increases. So we have also found the connection for computing decimals of the number Pi.⁷

5.Conclusion

When the first computers arrived, everything changed, Well, the way and possibilities in calculating the number Pi. With them, the records in the number of calculated digits are achieved every day by the Ghana. Billions are already far exceeded. On computers with the Microsoft Windows Vindous operating system, the PiFast program can be used to quickly calculate a large number of digits. The highest number of digits π calculated on the home computer is 25 000 000, for which PiFast took 17 days. To date, even over 1.25 trillion decimals of the number π have been calculated and this was done by the Japanese Yasumas Canada in 2002. In this paper, using the Mathematica software, it is suggested in the calculation of the number of Pi to 10 million decimal numbers. There are several tens of algorithms for calculating the decimals of Pi. Why was it interesting to scientists? What is the mystery of Pi? Knowing that the number Pi is infinitely large and the development of technology, and therefore computer systems as well as an increasing number of mathematical software and reaching 1.25 trillion decimals of the number Pi, scientists have come to the realization that every birth date can be entered by the appropriate algorithm as let a decimals of the Pi number. By typing the date of birth into the numerator, we get the position of the decimals on which our birth date is. This is just one of the points of interest. It can also be concluded that since the beginning of calculating the number of pi through historical development, the speed of arriving to a greater number of decimals is increasing. We can conclude that the development of the algorithms that included the geometric lines of the number pi could be calculated with a very large number of decimals, is a queue with a higher degree of convergence that is summarized in summation algorithms for solving directly linked to the number of digits of the number pi. Taking into account the aforementioned and interesting facts about the number of pi, there are even different types of arts that magnify the number Pi - pi - music, pi - poetry, pi - film ... Also, how much he has been intriguing to the public for centuries, the best shows are celebrations in his honor. Namely, every March 14 celebrates even the day of the number π (then Albert Einstein was born). Otherwise, the day of the Pi number is marked in various ways, including competition in eating pies or shooting pit and discussing the significance of the number π . Some schools also hold competitions in which the student will be able to remember the most decimals of the number Pi

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