

On inner contradiction in the metric tensor of the standard cosmological model

Iurii Kudriavtcev

The metrics in the basis of the standard cosmological model correspond to the absence of the energy exchange between the matter and the gravitational field which is equivalent to the impossibility of the development of the Universe in time. This way there is a deep inner contradiction in the basis of the standard cosmological model describing the developing Universe.

This contradiction is eliminated when we use the metrics that consider the non-zero value of the differential of the scale factor. The modified cosmological model built on basis of this metrics corresponds to the energy conservation law and describes the Universe that stays closed at any matter density and does not require introducing some additional non-observed substances (cosmological constant or dark energy), infinite in time and expanding in the accelerated manner on this stage.

Slower dynamics of the expansion eliminates some significant problems of the standard cosmological model related to the limits in time and also allows interpreting the recently discovered microwave background symmetry as a natural phenomenon, not related to the violation of the isotropy and other basic principles of the relativity theory.

98.80.k

1. Introduction. Inner contradiction of the standard cosmological model.

The equations of the general relativity theory represent the mathematical expression of the energy conservation law and Einstein pointed that out several times in his "Basics of the general relativity theory" [1]. The energy conservation law that concerns the matter energy and the energy of the gravitational field, in the general relativity theory is expressed by the tensor equation [2]:

$$(1/\sqrt{-g}) \partial T_i^k / \partial x^k - (1/2) \partial g_{ij} / \partial x^i T^{kl} = 0; \quad (1)$$

The second term in the left part of this equation represents an expression for the momentum and correspondingly for the energy that are transmitted in a unit of time and a unit of volume to the matter from the gravitational field [1]. Its zero component corresponds to the spatial density of the energy transmission.

Inserting to it the components of the metric tensor and energy-momentum tensor of the standard cosmological model, we will obtain that it is equal to zero (as all the components of T^{kl} are equal to zero except T^{00} , and $g_{00}=1=\text{const}$, from where $\partial g_{00} / \partial x^i = 0$), which corresponds to the absence of the energy exchange between the matter and the gravitational field and consequently to the impossibility of the development of the Universe within time. This way in the basis of the standard cosmological model describing the expanding Universe there is a deep inner contradiction that cannot be eliminated without changing the beliefs about the metrics.

2. Potential and kinetic energy of the bodies in Friedman's standard model.

Metrics in the basis of the standard cosmological model [2], is expressed by the correlation:

$$ds^2 = c^2 dt^2 - a^2 d\chi^2 + \sin^2 \chi (\sin^2 \theta d\varphi^2 + d\theta^2); \quad (2)$$

where a – radius of the curvature of the space (scale factor), χ – distance coordinate, θ, φ – angular coordinates, c – light speed. Corresponding values of the components of the covariant metric tensor: $g_{00} = 1, g_{11} = -a^2, g_{22} = -a^2 \sin^2 \chi, g_{33} = -a^2 \sin^2 \chi \sin^2 \theta$.

Expression (2) corresponds to the uniformly curved space obtained by Einstein by deriving the 4th spatial coordinate and its further exclusion through the radius of the space curvature [3]. This mathematical formalism that allows describing the curvature of the 3-dimensional space by the gravitational fields was, obviously, introduced by Einstein when studying the stationary Universe. When obtaining the expression (2) the differential of the excluded 4th spatial coordinate, being the part of the expression for the element of the spatial distance dl , is expressed by the differentials of the three other spatial coordinates [2], but not through the differential of the radius of the space curvature da , which is equal to zero in the stationary Universe:

$$d^2 = dx_1^2 + dx_2^2 + dx_3^2 + (x_1 dx_1 + x_2 dx_2 + x_3 dx_3)^2 / (a^2 - x_1^2 - x_2^2 - x_3^2). \quad (3)$$

Let us study the relation of the metrics (2) with the value of the gravitational potential φ , defining the value of the potential energy U_g of the body with the mass m in the gravitational field.

By the definition of the gravitational potential

$$U_g = m\varphi. \quad (4)$$

From the general relativity theory equations it follows [2], that in case of small speeds of the matter movement the gravitational potential is related to g_{00} by the expression:

$$g_{00} = (1 + 2\varphi/c^2). \quad (5)$$

But in the metrics (2) $g_{00} = 1$, from where it follows that this metrics in the basis of the standard cosmological model corresponds to the zero values of the potential energy of the material bodies in the gravitational field of the Universe. This way the potential energy of the bodies in the gravitational field in the standard model is excluded from the examination.

At that the material objects of the the Universe are considered in the accompanying coordinates which is reflected in the equations a zero speeds of the material objects [1,2]. The necessity of the choice of such coordinates is related to the condition of the space isotropy, equivalent to the absence of the separated directions which is realized at zero module of the velocity vectors. But satisfying to the condition of the isotropy, the accompanying coordinates exclude the kinetic energy of the expansion from the energy term of the equations, without compensating it by the corresponding changing in other terms. This way the kinetic energy of the body movement related to the expansion of the Universe is also excluded from the study.

3. Metric tensor with consideration of the non-zero differential of the scale factor in the expanding Universe

Let us see how the expression for the interval (2) will change when considering the dependency of the scale factor from time $a(t)$. Introducing analogically to [2][3] a notion about four-dimensional Euclidean space and expressing the fourth spatial coordinate through the differentials of the three other spatial coordinates and the differential of the curvature radius of the space da , which is for $a(t) \neq \text{const}$ not equal to zero, we will obtain the expression for the element of the spatial distance dl looking like:

$$dl^2 = dx_1^2 + dx_2^2 + dx_3^2 + (a da - x_1 dx_1 - x_2 dx_2 - x_3 dx_3)^2 / (a^2 - x_1^2 - x_2^2 - x_3^2); \quad (6)$$

where x_1, x_2, x_3 are Cartesian spatial coordinates. Moving from the Cartesian coordinates to the polar r, θ, φ and examining for simplicity only radical movements ($\theta = 0, d\theta = 0$), we will obtain

$$dl^2 = dr^2 + (da - (r/a)dr)^2 / (1 - (r/a)^2); \quad (7)$$

let us introduce analogically to [2], a coordinate χ from the expression $r = \sin(\chi)$. Then

$$dl^2 = da^2 + a^2 d\chi^2; \quad (8)$$

$$ds^2 = c^2 dt^2 - dl^2 = c^2 dt^2 - da^2 - a^2 d\chi^2 = c^2 dt^2 (1 - da^2/c^2 dt^2) - a^2 d\chi^2; \quad (9)$$

Обозначив $da/cdt = a'$, окончательно получим:

$$ds^2 = c^2 dt^2 (1 - a'^2) - a^2 d\chi^2; \quad (10)$$

This way the consideration of the dependency $a(t)$ gives the expression for the interval where the constant value of the component of the metric tensor $g_{00} = 1$ is changes to the variable

$$g_{00} = (1 - a'^2). \quad (11)$$

Putting to (2), we will finally obtain:

$$ds^2 = c^2 dt^2 (1 - a'^2) - a^2 [d\chi^2 + \sin^2 \chi (\sin^2 \theta d\varphi^2 + d\theta^2)]; \quad (12)$$

4. Law of mechanical energy conservation in the metric tensor of the expanding Universe.

Closed Universe in the standard cosmological model represents an expanding 3-dimensional hypersphere in a 4-dimensional Euclidean [4] space. In this space the notions of length (the hypersphere radius is defined) and time are defined. So, a consequent applying of this formalism requires to expand to it the notions of the velocity and the kinetic energy, Einstein's relativity principle and the expressions of the special theory of relativity.

Comparing (5) and (11), we obtain

$$a'^2 = -2\varphi/c^2; \quad (13)$$

$$\varphi = -(1/2)a^2/c^2; \quad (14)$$

Considering that $a' = da/cdt$

$$\varphi = -(1/2)\chi(da/dt)^2; \quad (15)$$

The derivative da/dt — is the linear velocity of the hypersphere expansion where any material body of the Universe moves in a 4-dimensional Euclidean space in a direction that is perpendicular to all the coordinate axis of the 3-dimensional space. Let us denote it as V_4 and considering (4) we will obtain:

$$U_g = -m V_4^2/2. \quad (16)$$

But $mv^2/2$ — non-relativistic (for the case of the small velocities which includes the expression (5) for the gravitational potential), expression for the kinetic energy E_k of the movent of the body with the mass m (in this case it is movement in 4-dimensional space), from where

$$U_g = -E_k. \quad (17)$$

This expression represents the mechanical energy conservation law that acts equally for all the material bodies in the Universe. In the point of the maximal expansion when $E_k=0$, U_g also becomes zero, in other moments the potential energy of the body in the gravitational field is negative and numerically equal to its kinetic energy of movement in 4-dimensional space with the opposite sign. Full mechanical energy W_m in any moment is equal to zero

$$W_m = U_g + E_k = 0, \quad (18)$$

which expresses the energy conservation law that represents in this formulation the law of development of the closed Universe. It is natural to expect that this law is also being realized at bigger velocities of movement, i.e. far from the point of the maximal expansion of the Universe but with the transition to the relativistic expressions for the kinetic and potential energy.

Let us note that it does not require to refuse from the accompanying coordinates that insure the requirement of isotropy of the space, as the movement of the objects takes place in the direction that is perpendicular to all the coordinate axis of the 3-dimensional space, and the velocities of the objects in this space can be still considered equal to zero.

This way the use in the model of the expanding Universe of the metrics that do not consider the dependency of the scale factor from time leads us to the exception of the kinetic and potential energies of the bodies from the study, and the consideration of this dependency includes them to the study and gives a simple expression for the law of conservation of the mechanical energy that is related to any material object in the Universe.

5. Realization of the energy conservation law of the general relativity theory

Expression (18) allows to calculate the velocity of energy transmission of the gravitational field (U_g) of the matter (E_k) into the unit of time in the unit of volume (D_E) and compare it to the non-zero component of the tensor equality (1).

$$D_E = d(U_g)/dt = -d(E_k)/dt = -d/dt[\mu V_4^2/2]; \quad (19)$$

where μ — average density (mass in the unit of volume),

To make it more simple let us write down (1) as:

$$A_{00} - B_{00} = 0; \quad (20)$$

where for the homogenous isotropic Universe in the corresponding coordinates

$$A_{00} = (1/\sqrt{-g}) \partial T_0^0 / \partial x^0; \quad (21)$$

$$B_{00} = (1/2) \partial g_{00} / \partial x^0 T^{00}. \quad (22)$$

$$x^0 = ct, \sqrt{-g} = \sqrt{(1-a^2)a^3 \sin^2 \chi \sin \theta}.$$

Non-zero components of the mixed and contravariant tensors of the energy-momentum accordingly to the expressions for the energy-momentum tensor of the solid macroscopic bodies [1], considering that the pressure can be taken as equal to zero, are defined by the expression:

$$T_0^0 = \varepsilon(dx^i/ds)(dx^0/ds) = \varepsilon; \quad (23)$$

$$T^{00} = \varepsilon(dx^0/ds)(dx^0/ds). \quad (24)$$

For the immobile object in the corresponding coordinates $dx^1=dx^2=dx^3=0$ and $ds = d(ct)\sqrt{(1-a^2)}$; (25)

from where $T^{00} = \varepsilon / (1-a^2)$. (26)

Putting (23)(26) to (21)(22), considering $\chi = \theta = \text{const}$ and $\partial x^0 \equiv \partial(ct)$ we will obtain

$$A_{00} = (1-a^2)^{1/2} a^3 \partial/\partial(ct) \{ \varepsilon (1-a^2)^{1/2} a^3 \}; \quad (27)$$

$$B_{00} = (1/2) \varepsilon (1-a^2)^{1/2} \partial/\partial(ct) (1-a^2); \quad (28)$$

Density of the energy ε in the expanding closed Universe depends from time and related to its full mass M by the expression

$$\varepsilon = \mu c^2 = Mc^2/V_{cl} = Mc^2/2\pi^2 a^3. \quad (29)$$

Putting (29) into (27)(28), and considering the mass of the Universe M to be a constant value that can be taken out of the sign of the derivative, we will obtain

$$A_{00} = (Mc^2/2\pi^2) (1-a^2)^{1/2} a^3 \partial/\partial(ct) (1-a^2)^{1/2}; \quad (30)$$

$$B_{00} = (1/2) (Mc^2/2\pi^2 a^3) (1-a^2)^{1/2} \partial/\partial(ct) (1-a^2); \quad (31)$$

Differentiating and simplifying we will finally obtain

$$A_{00} = - (Mc^2/2\pi^2) (1-a^2)^{1/2} a^3 a' a''; \quad (32)$$

$$B_{00} = - (Mc^2/2\pi^2) (1-a^2)^{1/2} a^3 a' a'' = A_{00}; \quad (33)$$

So, the energy conservation law in definition (1) at $g_{00} = (1-a^2)$ is being realized.

6. Comparison of the velocities of the energy transmission

Now we will calculate D_E . Putting to (18) μ from (29), we will obtain

$$D_E = - d/dt [\mu V_4^2/2] = - d/dt [(M/2\pi^2 a^3) (c^2 a^2/2)] = - (Mc^2/2\pi^2) d/dt [a^3 a^2/2] = - (Mc^2/2\pi^2) c d/d(ct) [a^3 a^2/2] = - (Mc^2/2\pi^2) c [a^3 a' a'' - (3/2) a^4 a'^3]; \quad (34)$$

As (18) is derived for the extreme case of the small velocities assuming that $a' \rightarrow 0$, we will neglect the second addend and will finally obtain:

$$D_E = - (Mc^2/2\pi^2) c [a^3 a' a'']; \quad (35)$$

which coincide with (32)(33) accurate to multiplier c , which had appeared at the transmission from the variable t to $x^0=ct$.

Let us check by dimensionality. The dimensionality of the energies transmitted in the unit of volume at the unit of time [$\text{kg}^1 \text{m}^1 \text{c}^3$], which coincide to the dimensionality of the right part (35). Non-coincidence with (32)(33) is obviously explained by the fact that these expressions were obtained for the unit of time (ct) in the assumption $c=1$. Considering this and assuming that the coincidence between B_{00} and D_E is actual not only for $a' \rightarrow 0$, we will obtain:

$$A_{00} = B_{00} = D_E = - (Mc^3/2\pi^2) (1-a^2)^{1/2} a^3 a' a''; \quad (36)$$

So, the metrics of the homogenous isotropic expanding Universe obtained with the consideration of the dependency of the scale factor from time (12), satisfies to the energy conservation law of the general relativity theory (1), and the velocity of the transmission of the energy of the gravitational field of the matter at the unit of time and in the unit of volume, following from (2), corresponds to this value in the energy conservation law (1).

7. Derivation of the field equations.

To derive the field equations that correspond to the metrics (12) with the variable value g_{00} , let us use the technique that we previously used in [2] to obtain the field equations of the standard cosmological model in the parametric form where instead of time we use the parameter η , that is defined by the correlation:

$$c dt = a d\eta. \quad (37)$$

Here t - time, c — light speed, a — scale factor, value of the time component of the metric tensor $g_{00} = a^2$.

If we write down the correlation (37) as:

$$c dt = b d\eta; \quad (38)$$

где b — a certain variable value that has, the same way the scale factor does, a commensuration of the length but generally speaking not equal to a , we will obtain $g_{00} = b^2$, and calculation of the components of the Ricci tensor for Einstein equations solution in the closed model of the Universe the same way as in [2] gives:

$$R_{00} = (3/ab)(a'b' - a''b); \quad (39)$$

$$R_{11} = a''a/b^2 + 2a^2/b^2 - aa'b'/b^3 + 2; \quad (40)$$

$$R_{22} = (a''a/b^2 + 2a^2/b^2 - aa'b'/b^3 + 2) \sin^2 \chi; \quad (41)$$

$$R_{33} = (a''a/b^2 + 2a^2/b^2 - aa'b'/b^3 + 2) \sin^2 \chi \sin^2 \theta; \quad (42)$$

$$R = (-6/a^3)(a + a''a^2/b^2 + a^2a/b^2 - a'b'a^2/b^3). \quad (43)$$

Inserting (39)(43) to the Einstein equation in mixed components and simplifying we are obtaining the same way as in [2] the field equation of the closed Universe in the form

$$a^2/b^2 - T_0^0 (8\pi G/3c^4) a^2 + 1 = 0; \quad (44)$$

where $a' = da/d\eta$.

If in (39)(44) we suppose $b=a$, they coincide with the obtained in [2] expressions for the standard cosmological model in parametric representation. If we suppose $b(t)=\text{const}=1$, $d\eta=cdt$, we will obtain the metrics (2) and equation (44) will represent

the dependency of the scale factor from the time for the standard model not in parametric but in the explicit form. Inserting $T_0^0 = \epsilon$, we will obtain an equation for the closed Universe

$$a^2 - (8\pi G\epsilon/3c^4)a^2 + 1 = 0; \quad (45)$$

where G — the gravitational constant, $a' = da/cdt$. Equation for the open model is different only by the sign of the free term:

$$a^2 - (8\pi G\epsilon/3c^4)a^2 - 1 = 0. \quad (46)$$

NOTE ABOUT THE OPEN MODEL: Open model unlike the closed one, describes the Universe of the infinite space which gives it some additional contradictory which is related to the fact that in the infinite space we can randomly separate however many areas that satisfy to Schwarzschild self force condition, i.e. closed ones which is contradictory to the open character of the model. In this relation we further examine only closed model of the Universe. We will show that when building the model on basis of the metrics (12) the Universe stays closed at any matter density.

Expression (44) allows us to obtain the equation that describes the dynamics of the development of the Universe and for $b(t) \neq \text{const}$. Let us insert the corresponding to (12) value $b^2(t) = (1-a^2)$.

$$a^2/(1-a^2) - T_0^0(8\pi G/3c^4)a^2 + 1 = 0; \quad (47)$$

Let us express T_0^0 thru ϵ from (23), and ϵ - through the volume of the space and mass of the Universe M according to (29). Inserting to (47), we will obtain the field equation in the form

$$2a_0/a = 1 + a^2/(1-a^2); \quad (48)$$

where the constant

$$a_0 = 2GM/3\pi c^2. \quad (49)$$

Having introduced the relative value of the scale factor $\alpha = a/2a_0$, we will obtain

$$a' = da/cdt = (1-\alpha)^{1/2}; \quad (50)$$

from where

$$dt = (2a_0/c) d\alpha / (1-\alpha)^{1/2}. \quad (51)$$

8. Question of the systems of references

When studying the Universe as an expanding hypersphere in the 4-dimensional Euclidean space where the ideas about the velocity and the equations of the special relativity theory are actual, we can use two systems of references:

- the coordinates related to the mass center of the Universe located in the geometrical center of the hypersphere,
- the coordinates of the observer located in some point of the surface of the hypersphere and immobile in relation to this point.

As these coordinates at any moment of time are moving in relation to each other with relative velocity $\beta = v/c = a'$, the velocities of the time streams in them are different. Which one is related to the time t included to the field equation?

If t is the time in the observer's coordinates on the surface of the hypersphere (we will denote it as K), then the own time t_c of the hypersphere center that moves away from the observer with the relative velocity $\beta = a'$, will be defined by the expression of the special relativity theory

$$dt_c = dt (1 - \beta^2)^{1/2} = dt / \gamma; \quad (52)$$

where

$$\gamma = (1 - a'^2)^{-1/2}. \quad (53)$$

However it has no any practical sense as the observation and the measurements take place in the same coordinates K , which is related to all the equations that describe the Universe. Let us note that the contradictory mentioned above is common to this variant.

In the opposite case time t which is included to the field equations is related to the coordinates of the center of the hypersphere (mass center) K_c , which is related to the clock that moves away from the observer on the surface of the expanding hypersphere with relative velocity $\beta = a'$. In this case t is own time of the moving clock in the mass center related to the time t_{obs} in the observer's coordinates by the expression

$$dt = dt_{obs} / \gamma. \quad (54)$$

Let us note that with such an approach the accompanying system of references becomes 4-dimensional and, still expanding with the 3-dimensional space — surface of the hypersphere, gets the central point, looking at which we go from the 3-dimensional accompanying system of references that excludes the kinetic energy of the expansion from the examination to the inertial system of references that is immobile in relation to the mass center. At that we include to the examination the expansion energy as all the matter moves in relation to these coordinates with the velocity $\beta = a'$. The examined above manifestation of the mechanical energy conservation in the metric tensor confirms the righteousness of these coordinates as the potential energy of the matter in the gravitational field is numerically equal (with the sign “-”) to its kinetic energy exactly in the mass center coordinates.

At that the condition of the isotropy of the 3-dimensional world is kept as the expansion takes place in the direction perpendicular to all of its spatial coordinates.

9. Possibility of consideration of the kinetic energy of expansion.

In the system K_c the mass of every material element of the Universe will be defined by an expression of the restricted relativity theory

$$m' = m(1 - \beta^2)^{-1/2}; \quad (55)$$

where β — its relative speed. But all the objects in the Universe move relatively to the center of the hypersphere with the same module of the relative speed equal to a' , that is why the formula (55) can be related to the mass of the Universe in general:

$$M' = M(1 - a'^2)^{-1/2} = \gamma M. \quad (56)$$

Then the mass M in the field equation must be changed to $M' = \gamma M$, which is the equivalent to the change of the constant a_0 the equation (48) by the product γa_0 . Having done this change and considering that the denominator of the expression in the right part can also be expressed through γ , we will obtain:

$$(2a_0/a)^2 = \alpha^2 = \gamma^2; \quad (57)$$

from where, inserting (53), we will obtain the equation that defines the dynamics of the development of the Universe for the metrics (12), in the form:

$$a' = da / cdt = (1 - \alpha^2)^{1/2}; \quad (58)$$

from where

$$dt = (2a_0/c) d\alpha / (1 - \alpha^2)^{1/2}. \quad (59)$$

10. Expression of the velocity of the expansion through the observed values.

Let us connect the velocity of the expansion of the hypersphere a' with the observed values. The velocity of the objects moving away depending from the distance is given by Hubble constant which is defined as

$$H = (1/a)(da/dt). \quad (60)$$

Having expressed a through α and da/dt through a' , we will obtain:

$$H = (c/2a_0\alpha) a'; \quad (61)$$

$$a_0 = 2GM/3\pi c^2 = (2G/3\pi c^2) 2\pi^2 a^3 \mu = (2G/3\pi c^2) 2\pi^2 (2a_0\alpha)^3 \mu; \quad (62)$$

where μ — the matter density. Having divided both parts to a_0^3 and transforming, we will obtain the equality

$$(c/2a_0\alpha) = (8\pi G\mu/3)^{1/2} \alpha^{1/2}. \quad (63)$$

Inserting to (61), we will obtain

$$H = (8\pi G\mu/3)^{1/2} \alpha^{1/2} a'; \quad (64)$$

from where

$$a' = \alpha^{1/2} (3H^2/8\pi G\mu)^{1/2}. \quad (65)$$

Noting that $(3H^2/8\pi G)$ represents an expression for the critical density μ_c , and introducing the relative density

$$\Omega = \mu / \mu_c; \quad (66)$$

we will obtain the expression that relates the velocity of the expansion a' with the observed values H and μ , expressed through the relative density Ω , as

$$a' = \alpha^{1/2} \Omega^{1/2}. \quad (67)$$

11. Equation for the main parameters of the closed Universe

Inserting (67) into (58), we will obtain the expression

$$\Omega = \alpha^4 (1 - \alpha^2)^{-1}, \quad (68)$$

allowing to find from the observed values H and μ , included to Ω , the relative value of the scale factor α and further from (63) the value of the constant a_0 , i.e. main geometrical parameters of the Universe.

However in the closed model $0 < \alpha < 1$, from where the right part of this equation is always > 1 , while the left part i.e. Ω , accordingly to the observed data is < 1 . At that the equation (68), as well as the analogical equation for the standard model

$$\Omega = (1 - \alpha)^4; \quad (69)$$

does not have actual solutions.

This contradiction between the observed matter density and the equality (68) requires further study. As it is related to the value of the critical density μ_c , let us see is its value can be influenced by the made before conclusions about the reference system. The expression for μ_c includes two physical values — gravitational constant G and Hubble constant H .

12. Checking the influence of the gravitational constant.

Einstein's relativity principle requires independency from the system of references of one of the world constants — light speed, but does not put the same requirements on the other values considered to be world constants, including the gravitational constant whose constancy or inconstancy is being discussed for more than half a century.

Let us study this question from the point of view of the two system of references — the one of the observer on the surface of the hypersphere (K), and the one of the hypersphere center (K_c), moving away from the observer with relative velocity $\beta = a'$ in a direction perpendicular to all the three spatial coordinates.

We measure the gravitational constant in the observer's coordinates. If time t in the field equations is the time in the observer's coordinates, its value in the equations is equal to the measured one. But if the field equations are related to the hypersphere center coordinates we should check if the value of the gravitational constant can have in this coordinates, i.e. in the field equations, a value equal to the measured one.

For this let us examine in the coordinates K and K_c a simple system of the interacting bodies, for example, a body with the mass m , rotating around a planet with the mass M by a circular orbit with the radius R .

Condition of the equilibrium of the body on the circular orbit:

$$F_{\text{grav}} = GmM/R^2 = F_{\text{centr}} = m\omega^2 R; \quad (70)$$

where ω — angular velocity, from where

$$G = \omega^2 R^3 / M = 4\pi^2 R^3 / MT^2. \quad (71)$$

All the values related to K_c , will be marked by the index (c). As the plane surface of the rotation is perpendicular to the velocity of the mutual moving away of the system of references, $R_c = R$, from where $4\pi^2 R^3 = \text{const}$.

$$G = \text{const} / MT^2. \quad (72)$$

Let us examine the obtained expression from the point of view of the momentum conservation law ($\mathbf{M}_0 = [\mathbf{R} \times \mathbf{p}] = \text{const}$). For the system of two bodies with the masses m and M , rotating around the common center of the inertness by circular orbits

$$|\mathbf{M}_0| = M_0 = \omega R^2 (Mm/M+m); \quad (73)$$

where R — distance between the mass centers, ω — angular velocity of the rotation.

At $M \gg m$, considering (71), we will obtain

$$M_0 = \omega R^2 m = m(GMR)^{1/2}, \quad (74)$$

from where

$$G = M_0^2 / Mm^2 R = \text{const} / Mm^2. \quad (75)$$

Now let us see the correlations of all the values included to the obtained expressions in the systems of references K and K_c . As we already mentioned, $l_c = l$; $R_c = R$. Let us examine T , M , m .

Accordingly to the conclusion, made on the basis of the study of the question about the systems of references, we presume that the field equations are related to the coordinates K_c . The interval of the own time of this system dt is related to the interval of time dt_{obs} in the observer's coordinates K as $dt = dt_{\text{obs}} / \gamma$ (54). So, the period of time T , measured in the coordinates K , will be related to the value in the coordinates K_c by the correlation

$$T_c = T / \gamma; \quad (76)$$

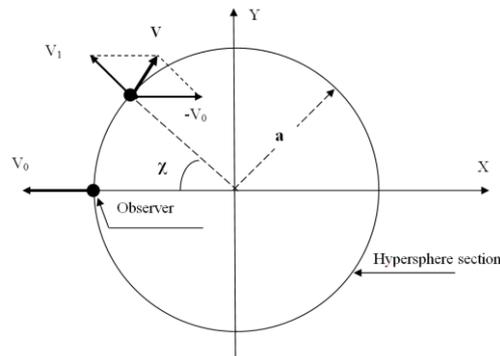
From comparison of (72) and (75) we obtain that the period T and the masses m , M in the examined examples change when transitioning from one system of reference to another by the same law, from where

$$M_c = M / \gamma; \quad (77)$$

$$G_c = \text{const} / M_c T_c^2 = \gamma^3 \text{const} / M T^2 = G \gamma^3. \quad (78)$$

13. Checking the influence of Hubble constant.

The velocity of the movement of the observed object in relation to the observer is equal to the difference of the vectors of velocity of expansion of the hypersurface in 4-dimensional space in the points of location of the observed object and the observer. Modules of these vectors for any point of the Universe neglecting bodies' own velocities have value $\beta = a'$, and the directions are perpendicular to the hypersurface. Since we examine only radial movements, ($\theta = \varphi = 0$), we can graphically show the model of the 4-dimensional space in 2-dimensional section (Picture 1).



Picture 1. Two-dimensional section of the 4-dimensional Euclidean space with hypersphere of the radius «a». Angular coordinate χ characterize the hypersurface point moving away from the observer (black spot). Vectors V_0 and V_1 - velocities of the movement of the observer and observed data. Vector V — velocity of the movement of the object in relation to the observer.

The sum of the vectors V_1 and $-V_0$ by the formulas of the relativistic adding up of the velocities gives the expression for the dependency of the moving away velocity V of the object from the coordinate χ

$$V^2(\chi) = c^2 \beta^2 (2 - 2 \cos(\chi) - \sin^2(\chi) \beta^2) / (1 - \beta^2 \cos(\chi))^2; \quad (79)$$

Going to the limit $\chi \rightarrow 0$, we obtain

$$V(\chi) \rightarrow c \beta \chi / (1 - \beta^2)^{1/2}; \quad (80)$$

Let us express the spatial distance D through the angular coordinate χ ($D = a \chi$) and considering that $\beta = a'$, we obtain:

$$V(D) = dD/dt = ca'(D/a)(1 - a'^2)^{1/2} = (ca'/a) D \gamma = H D \gamma; \quad (81)$$

where H — Hubble constant.

Let us denote the coefficient of the proportionality between the observed velocity of the objects moving away and distance to them as $H_{\text{obs}} = V_{\text{obs}}/D$ and express it through Hubble constant from (81) and considering (54):

$$H_{\text{obs}} = (1/D)(dD/dt_{\text{obs}}) = (1/\gamma D)(dD/dt) = (1/\gamma D) H D \gamma = H. \quad (82)$$

This way the values of Hubble constant in the observer's coordinates and the hypersphere surface are numerically equal and this magnitude does not influence the value of the critical density.

14. In which coordinates the gravitational constant is constant?

The gravitational constant is included to the initial field equations. We established that its value in the coordinates of the mass center of the Universe is different from the values in the immobile observer's coordinates and related to him by the coefficient γ^3 , depending from the velocity of the expansion i.e. changing within the time. That is why there is a question popping up — in which of these coordinates the gravitational constant is constant?

If it is constant in the observer's coordinates ($G = \text{const}$), it will be variable in the mass center coordinates ($G_c \neq \text{const}$), where the field equations are included. Then we should consider the variability of this value by making a change in the initial equation (57) $G \rightarrow G_c = G \gamma^3$.

Change $G \rightarrow G_c$ in the equation (57) will lead to the change $a_0 \rightarrow a_{0c} = a_0 \gamma^3$. Changing then from the variable a' to the variable γ and simplifying, we will obtain the equality:

$$\alpha = \gamma^2, \quad (83)$$

which is not realized at all the $\alpha \neq 1$. Thus way the assumption that the value G is constant leads us to the contradiction and is wrong.

Let us see the opposite variant ($G_c = \text{const}$, $G \neq \text{const}$). In this case the included to the equation value of the gravitational constant G_c stays constant and the equation (57) and the following from it dependencies (58)(59), defining the dynamics of the development of the Universe, stay unchanged. But the expression (68), that relates the measured value G with the current value of α , should include the depending on α correlation between G and G_c .

Let us insert G_c to the expression for μ_k ($\mu_k^c = 3H^2/8 \square G_c$) and express it through the measured value of G from (78). This leads to the change $\mu_k \rightarrow \mu_k^c = \mu_k / \gamma^3(\alpha)$.

Inserting to (68), we will obtain:

$$\Omega \gamma^3 = \alpha^3 (1 - \alpha^2)^{-1} \tag{84}$$

and considering (57)

$$\Omega = (1 - \alpha^2)^{-1} \alpha^{-2} \tag{85}$$

$$\alpha = \Omega^{1/2} (1 + \Omega)^{-1/2} \tag{86}$$

Equalities (85)(86) are actual not only at $\Omega > 1$, but also at $\Omega < 1$. Contradiction between theoretical correlation and observing data that is characteristic for the standard model and in this case is absent and the Universe stays closed at any relative density of the matter.

15. Dynamics of the development of the Universe in the observer's coordinates.

Let us see the dynamics of the expansion of the Universe in the immobile observer's coordinates. Time t is related to the time t_{obs} in the observer's coordinates by the expression (54). Inserting to the expression for dt (59), considering (57) we will obtain the expression for $dt_{\text{obs}}(\alpha)$:

$$dt_{\text{obs}} = (2a_0/c) d\alpha / [\alpha (1 - \alpha^2)^{1/2}] \tag{87}$$

$$(d\alpha/dt_{\text{obs}}) = (c/2a_0) \alpha (1 - \alpha^2)^{1/2} \tag{88}$$

$$H = (1/a) (da/dt_{\text{obs}}) = (1/\alpha) (d\alpha/dt_{\text{obs}}) = (c/2a_0) (1 - \alpha^2)^{1/2} \tag{89}$$

considering (86), we will obtain expressions for a_0 и a :

$$a_0 = (c/2H) (1 - \alpha^2)^{1/2} = (c/2H) (\Omega + 1)^{1/2} \tag{90}$$

$$a = 2a_0 \alpha = (c/H) \Omega^{1/2} (\Omega + 1)^{-1} \tag{91}$$

The values of the scale factor for values of the average density $0,05 < \Omega < 0,5$ calculated by the formula (91) lay within the range from 3 to 7 billions of light years.

Integrating (59) and (87), we will obtain analytical expressions for $t(\alpha)$ and $t_{\text{obs}}(\alpha)$, and reverse to them $\alpha(t)$ and $\alpha(t_{\text{obs}})$, defining the dynamics of the expansion in the coordinates of the hypersphere center and in the observer's coordinates:

$$\tau(\alpha) = \arcsin(\alpha) + C; \tag{92}$$

$$\alpha(\tau) = \cos(\tau); \tag{93}$$

$$\tau_{\text{obs}}(\alpha) = \ln(\alpha / (1 + \sqrt{1 - \alpha^2})) + C; \tag{94}$$

$$\alpha(\tau_{\text{obs}}) = 1 / \text{ch}(\tau_{\text{obs}}); \tag{95}$$

where

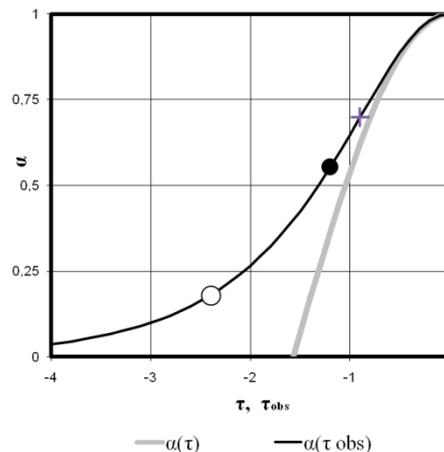
$$\tau_{\text{obs}} \equiv (c/2a_0) t_{\text{obs}} \tag{96}$$

$$\tau \equiv (c/2a_0) t \tag{97}$$

Counting τ and τ_{obs} is done from the moment of the maximal expansion ($\alpha = 1$).

Dependencies $\alpha(\tau)$ and $\alpha(\tau_{\text{obs}})$ graphically shown on the Picture 2. On the curvature $\alpha(\tau_{\text{obs}})$ we mark the points of the current condition of the Universe for the density which is approximately equal to the density of the visible matter ($\Omega = 0,03$) – light circle and for the maximal value of the density considering invisible dark matter ($\Omega = 0,4$) – dark circle. The point of bend is marked with the cross.

Expansion in the observer's coordinates takes place by the law of the hyperbolic cosecant, time of the life of the Universe in these coordinates happens to be infinite and the values of the average density of the matter obtained from the observing data correspond to the closed model of the Universe on the stage of the accelerated expansion (lower than the point of the bend on this curvature at $\alpha = 1/\sqrt{2}$).



Picture 2. Dynamics of the expansion of the closed Universe with consideration of the dependency of the scale factor from time ($g_{00} = \gamma^2$) in different coordinates. Time is expressed in relative units $\tau \equiv (c/2a_0)t$ and is counted from the moment of the maximal expansion ($\alpha = 1$). On the curvature $\alpha(\tau_{\text{obs}})$ we mark the positions of the current condition of the Universe at $\Omega = 0,03$ (light circle) and at $\Omega = 0,4$ (dark circle). The point of bend is marked with the cross.

16. Main particularities of the modified model

The creation of the model of homogeneous isotropic expanding Universe on basis of the metrics that consider the dependency of the scale factor from time brought us to the modified cosmological model corresponding to the energy conservation law and describing Universe which is closed at any matter density and does not require inserting of the additional non-observed substances (cosmological constant, dark energy, vacuum energy), infinite in time and expanding in the accelerated manner at this moment which corresponds to the observation data about the accelerated expanding [5].

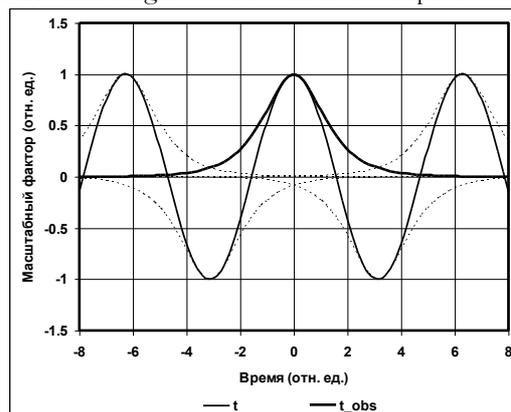
Expanding in the observer's coordinates, unlike the one in the standard model, happens slowly which eliminates the problems in the basis of the scenario of the expanding Universe [6]: "horizon problem", "singularity problem", "flatness problem" (formulated as a question about the reasons of the accurate corresponding of the matter density in the early Universe to the critical value and is being the basis of the "anthropic principle") - in the modified model disappears, as at $\alpha \rightarrow 0 \Omega \approx \alpha^2$, "problem of the large-scale homogeneity and isotropy" is related to the "horizon problem", "relict monofield problem" and "relict gravitino problem" - also defined by the dynamics of the development, "vacuum energy problem" - solved in the modified model without inserting the additional substances, etc.

Recently discovered symmetry of the microwave background inhomogeneities is interpreted in the standard model as a possible evidence of the violation of the basic requirement of the space isotropy ("Axis of Evil") [7], but in the modified model it is interpreted as a natural phenomenon that follows from the slower dynamics of development and not related to the violation of the relativity theory principles. The time that passed from the moment of the radiation of the signal by the distant source, for example by the inhomogeneity of the matter density at the period of the hydrogen recombination, happens to be sufficient for that the radiation of the source, going in all directions, reached the observer not only by the small but also by the big arc of the big circle of the closed Universe, i.e. from the opposite side which is seen by the observer as a phenomenon of the central symmetry [8] — a possibility to observe the same source in two opposite (centrally symmetrical) points of the celestial sphere.

The phenomenon of the central symmetry is confirmed both by the central symmetry and antisymmetry of the microwave background [8] and by the existing of the centrally symmetrical pairs of quasars with the identical luminosity magnitude profiles in the ranges (u,g,r,i,z) [9], which can also be interpreted as pairs of the opposite images of the same distant object.

Except the particularities related to the interpretation of the observation data, the equations (93)(95), describing the dynamics of the development of the Universe in the coordinates of the observer and of the hypersphere center, allows to assume a possible variant of its previous history which is principally different from the variants based on the standard model. According to (93), in the mass center coordinates the scale factor changes by the law of $\cos(t)$, which means the Universe generally behaves as a classic harmonic vibrator. At that from the mechanical energy conservation law (17) we see that inside of it as inside any other harmonic vibrator, there is a constant transformation of the kinetic energy to the potential and vice versa. Such manner of changes can be considered a natural form of the manifestation of the philosophical thesis about the impossibility of existence of the matter without movement.

When there is a certain quantity of the matter (energy) and there is no external influences, this vibrator can oscillate eternally, the same way the classic harmonic loss-free vibrator does. On every half-period of the oscillations it brings to birth a new 3-dimensional space — hypersphere developing in its own system of references during an infinite time. An obtained "harmonic generator of the eternities" is graphically shown on the Picture 3. Every point of the infinite time axis in the mass center coordinates corresponds to the infinite quantity of the eternal spaces born on different half-periods of the vibrator, however all of them except one are related to the initial and final stages of the evolution of the space.



Picture 3. Dynamics of the development of the modified model of the Universe on the time scales of the hypersphere center (t) and observer in a 3-dimensional world (t_{obs}). Harmonic oscillations in the mass center system giving birth to the eternal Universes on every half-period of the oscillations.

17. Conclusion

General relativity theory is one of the most perfect creations of the modern physics. Its correctness is proved by many experimental and observation data but the extremal complexity of the theory requires some special attention to the notions and physical values used in it and increases the danger of the incorrect interpretation. It is related to the known contradictory [4] based on the general relativity theory of the Friedman's standard cosmological model.

In this work we show that the metrics in the basis of the standard cosmological model correspond to the absence of the energy exchange between the matter and the gravitational field defined by the energy conservation law of the general relativity theory which is equivalent to the impossibility of the development of the Universe within time. This way there is a deep inner contradiction in the standard cosmological model that describes the developing Universe.

Contradiction is eliminated when we use the metrics that consider the dependency of the scale factor from time i.e. the non-zero value of its differential that plays a significant role in the derivation of the expression for the time component of the metric tensor. In this case the energy-momentum conservation law in the general relativity theory is realized and the speed of the energy exchange of the general relativity theory happens to be equal to the energy exchange that follows from the analysis of the components of the metric tensor.

Building up the model of the homogenous isotropic Universe on basis of these metrics leads us to the modified cosmological model that corresponds to the energy conservation law and describes the Universe, closed at any matter density that does not require inserting the additional non-observed substances (cosmological constant or dark matter), infinite in time and expanding in accelerated manner at this stage. Slower dynamics of the expansion eliminates several problems of the standard model: “singularity problem”, “horizon problem”, “flatness problem”, etc., and also allows to interpret the recently discovered symmetry of the microwave background as a phenomenon that follows from the slower dynamics of the expansion and is not related to the violation of the fundamental principles of the relativity theory.

27 march 2011

Bibliography:

1. Albert Einstein. Basics of the general relativity theory. (*Die Grunlage der allgemeinen Relativitatstheorie, Ann. d. Phys., 49, 769 (1916)*). In “Albert Einstein and the gravitation theory. Articles”. Moscow, 1979., p. 146-189.
2. Landau L.D., Lifshitz E.M. Course of Theoretical Physics: The Classical Theory of Fields. Vol.2.)
3. Albert Einstein. “Questions of cosmology and the general relativity theory” (*Kosmologische Betrachtungen zur allgemeinen Relativitatstheorie. Sitzungsher preuss. Akad. Wiss., 1917, 1, 142-152*). In “Albert Eistein. Works.” Vol. 1. Moscow, 1965, p. 601-612.
4. Weinberg S., Gravitation and Cosmology: Principles and applications of the General Theory of Relativity, John Wiley and Sons, Inc., 1972).
5. Riess A G et al. Astron. J 116 1009 (1998); Perlmutter S et al. Astrophys. J 517 565 (1999).
6. Linde A, Expanding Universe. Physical Succeses. October 1984, p.177-214.
7. K. Land, J. Magueijo. Examination of Evidence for a Preferred Axis in the Cosmic Radiation Anisotropy. Phys. Rev. Lett. 95, 071301 (2005).
8. Iurii Kudriavtcev, Dmitry A. Semenov. Central symmetry and antisymmetry of the microwave background inhomogeneities on Wilkinson Microwave Anisotropy Probe maps. arXiv:astro-ph/1008.4085.
9. Iurii Kudriavtcev. Manifestation of central symmetry of the celestial sphere in the mutual disposition and luminosity of the Quasars. arXiv:astro-ph/1009.4424.