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Point interactions in quantum mechanics

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Abstract. *Models of point interactions that approximate the real interactions of atomic particles in quantum mechanics are considered. In a one-point one-dimensional system, states are studied taking into account the eigenfield and a non-stationary problem is solved. The concept of a "point cluster" is introduced - a complex of charges that, at zero size, create the possibility of localization of a test particle in a region of finite size. An integral model of point interaction is proposed. States in three-dimensional two-point systems with "local isotropy"are studied.*

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Introduction

Extensive literature is devoted to the use of point potentials for the model description of atomic processes – see, for example, [1], [2]. This circumstance is facilitated by the apparent simplicity of the description with the help of-potentials, associated with the possibility of analytical solution of complex problems. At the same time, however, the method of creating a system characterized by a zero interaction radius is usually not studied. It can be assumed that with a really small size of the positive charge, a single bound level plays a determining role in the behavior of the system, then the method of creating the potential is not essential. The works [3], [4] are devoted to a detailed theoretical study of the properties of point potentials. It is worth mentioning the works that consider the exact solutions of non-stationary problems of scattering potentials– [5], [6], [7], illustrating the possibilities of the considered model method. A complete self-consistent description of systems with point or delta potentials may be of interest from both theoretical and experimental points of view. In this paper, we study the interaction of point clusters in one-dimensional systems and the states of point systems in three-dimensional spherically symmetric systems. In single-point systems, the influence of the eigenfield was studied, and in the one-dimensional case, in addition, a non-stationary problem was considered.

1. One-dimensional quantum mechanical system with a point potential

Let us first consider a one-dimensional system described by a one-dimensional Schrodinger equation of the following form:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + \alpha\delta(x)\Psi \quad (1.1)$$

This equation has a stationary solution: $E = -\frac{\hbar^2\gamma^2}{2m}, \alpha = -\frac{\hbar^2\gamma}{m},$

$$E\psi = -\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \alpha\delta(x)\psi \quad (1.2)$$

Substituting this expression in (1.2) leads to the equalities:

$$\psi''(s) = (1 - 2\delta(s) + V(s)) \quad (1.3)$$

$E = -\frac{\hbar^2\gamma^2}{2m}, \alpha = -\frac{\hbar^2\gamma}{m}$. Next, we introduce a dimensionless variable $s = \gamma x$. Equation (1.2) takes the form:

$$\psi''(s) = (1 - 2\delta(s))\psi(s) \quad (1.4)$$

Next, we will investigate the effect of the eigenfield on the state of a charged system with a point potential. In the right part (1.3), the potential determined by the charge of the layer.

$$\psi''(s) = (1 - 2\delta(s) + V(s))\psi(s), V''(s) = -\kappa_*|\psi|^2 \quad (1.4)$$

Let's put $\psi = Z(s)\exp(-|s|)$, then the system takes the form:

$$Z" - 2\,Z'sigmum(s) = V(s)Z(s), V'' = -\kappa_0 Z^2\exp(-2|s|). \quad (1.5)$$

The initial values were taken during the calculations $Z'(0) = 0, Z(0) = 2, \kappa_0 = 10$. Figure 1 shows the graphs of the functions $\psi(s)$ and function $V(s)$. You can see that near the axis $S = 0$ $\psi(s)$ practically coincides with the exponential expression, whereas in the opposite case, the exponent practically vanishes, and $\psi(s)$ it becomes oscillatory in nature.

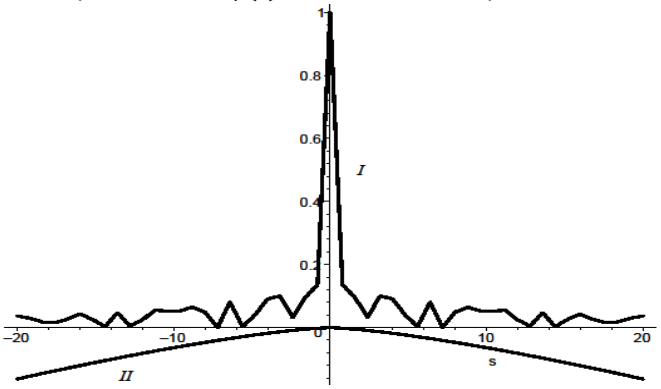

Fig. 1. **Dependence of the potential $V(s)$ (II) and $\psi(s)$ (I)**

2. Non-stationary system

The equation has the form:

$$i\hbar\frac{\partial\Psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} + \frac{\alpha}{\xi(t)}\Psi + \frac{1}{\xi^2}U\left(\frac{x}{\xi(t)}\right)\Psi \quad (2.1)$$

Where $\xi(t)$ – the specified function, $\ddot{\xi}(t) = \frac{\lambda}{\xi^3}$, λ- constant.

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Let's introduce new variables: $x_* = \frac{x}{\xi(t)}, \tau = \int \frac{dt'}{\xi(t')^2}$. We will get:

$$i\hbar\left(\frac{\partial\Psi}{\partial\tau} - x_*\frac{d\xi}{d\tau}\frac{1}{\xi}\frac{\partial\Psi}{\partial x_*}\right) = -\frac{\hbar^2}{2m}\frac{\partial^2\Psi}{\partial x^2} = \alpha\delta(x_*)\Psi. \quad (2.2)$$

Variables x_*, τ in (2.2) are separated, if $\frac{d\xi}{d\tau} \equiv const$.

We denote: $\frac{d\xi}{d\tau}\frac{1}{\xi} = \frac{1}{2\tau_9}$. You can get: $\xi = \sqrt{\xi_0^2 + \frac{t}{\tau_0}}$.

We present the solution (2.2) in the form: $\Psi = T(\tau)X(x_*)$. Then it follows from (2.2):

$$i\hbar\left(\frac{\dot{T}}{T} - \frac{x_*}{2\tau_0}\frac{X'}{X}\right) = \frac{\hbar^2}{2m}\frac{X''}{X} + \alpha\delta(x_*) \quad (2.3)$$

In (2.3), a stroke means a derivative of x_* and the point is the derivative of τ.

Let's put: $T(\tau) = \exp\left(-\frac{iE\tau}{\hbar}\right)$, where E – the actual value. From (2.3) follows the relation:

$$E = i\hbar\frac{x_*X'}{2\tau_0 X} - \frac{\hbar^2}{2m}\frac{X''}{X} + \alpha\delta(x_*) \quad (2.4)$$

Let's put: $R = Z\exp(-|s|)$ then when $E = -\hbar\tau_0, \gamma = -2$ we get the system:

$$Z'' - 2Z'signum(s) - \frac{s^2}{16}, Y' = -\frac{1}{4} + 2Y\frac{Z'}{Z} - signum(s). \quad (2.5)$$

This system was solved with boundary conditions: $z(0) = 1, Z'(0) = 0, Y(0) = 0$. The results of the solution are shown in Fig.2. A non-stationary solution is considered that does not take into account its own field, here the charge density depends only on the self-similar variable s.

Which makes it impossible to derive a self-consistent equation for the field. To output a self-consistent system, it is necessary to use another method of separating variables (see [8]).

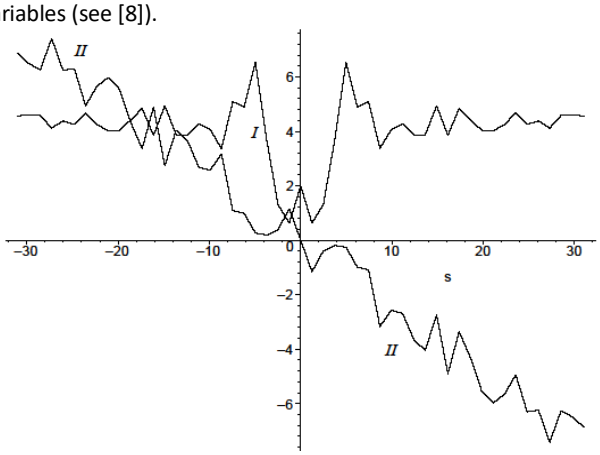

Fig. 2. **Dependences $\psi(s)$ (I) and $Y(s)$ (II)**

3. Interaction of clusters

Let's consider how third-party charges can create a potential $V = -\alpha\delta(x)$ if the interaction is electromagnetic. Let's use the Poisson equation written in dimensionless variables: where ρ - the linear charge density of the object that creates δ - the potential, this object will be further referred to as a "cluster". The cluster charge density satisfies the relation: $\rho = -\frac{\alpha}{4\pi}\delta''(x)$. This equality means that the full charge of the cluster q is equal to zero - $\int_{-\infty}^{+\infty} dx\rho(x) = 0$, the dipole moment is also zero - $d_1 = \int_{-\infty}^{+\infty} xdx\rho(x) = 0$ and only the quadratic moment is different from zero: $d_2 = \int_{-\infty}^{+\infty} x^2 dx\rho(x) = \frac{\alpha}{2\pi}$. It follows from the above considerations that in order to hold a charged particle (electron) in a limited region, it is not necessary to have a charge of the opposite sign in the center of the system – an electron can be localized if it interacts with a point cluster characterized by a zero charge, but having a quadratic moment other than zero. In this case, the cluster field is everywhere except for the neighborhood of the point $x = 0$ is equal to zero.

Consider a situation where there are two δ - the center, i.e. the potential has the form:

$$V = -\alpha_1\delta(x - x_0) - \alpha_2\delta(x + x_0) \quad (3.1)$$

Solution of the stationary Schrodinger equation characterized by the depth of the level $\frac{\beta^2}{2}$ you can search in the form:

$$\psi = a\exp(-\beta|x - x_0|) + b\exp(-\beta|x + x_0|) \quad (3.2)$$

Substituting (3.2) into (1.1), taking into account (3.1), we obtain the system:

$$a\beta = \alpha_1\left(a + b\exp(-2\beta x_0)\right) = \alpha_2(\exp(-2\beta x_0) + b) \quad (3.3)$$

Where a, b – constants. From the condition of having a non-zero solution of this system for a, b for the value β you can get the ratio:

$$(\beta - \alpha_1)(\beta - \alpha_2) = \alpha_1\alpha_2\exp(-4\beta x_0) \quad (3.4)$$

In the case when $\alpha_1 = \alpha_2$ symmetric and antisymmetric solutions are possible for ψ – functions. If $a = b$ that $\alpha = \frac{\beta}{1+2\beta\exp(-2\beta x_0)}$, and if $a = -b$ that $\alpha = \frac{\beta}{1-\exp(-2\beta x_0)}$.

Let us further consider the effect of the field of an electron connected by two centers on these centers themselves – point clusters. The total force acting on the cluster is calculated as an integral of the following form:

$$F = \int dx\rho_{cl}E(x), \quad (3.5)$$

where ρ_{cl} – cluster charge density, $E(x)$ – the field created by the bound electron. This field is determined from the equation: $div\vec{E} = -4\pi\rho_e$ где $\rho_e = |\psi|^2$. That is, the field must satisfy the equation:

$$\frac{dE}{dx} = C_0^2|\exp(-\beta|x - x_0|) + \exp(-\beta|x + x_0|)|^2 \quad (3.6)$$

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This equation corresponds to a symmetric distribution of the electron density, C_0 – the normalization constant. Since the charge density of the cluster located at the point $x = x_0$ defined as $\rho_{cl} = \frac{\alpha}{4\pi}\delta''(x - x_0)$, the force acting on this cluster is determined by the equality:

$$F = \alpha\int dx'\delta''(x - x_0)E(x') = -\alpha\int dx'\delta'(x - x')E'(x') = -4\pi\alpha C_0^2 \int dx'\delta'(x - x')\big(\exp(-\beta|x' - x_0|) = \exp(-\beta|x' + x_0|)\big)^2.$$

The derivative of the integrand has a discontinuity at the point near which the cluster is localized:

$$\frac{d}{dx'}\big(\exp(-\beta|x - x'|) + \exp(-\beta|x' + x_0|)\big)^2 = -2\beta\big(\exp(-\beta|x' - x_0|) + \exp(-\beta|x' + x_0|)\big) \times$$
$$\big(\exp(-\beta|x' + x_0|)sigmum(x' - x_0) + \exp(-\beta|x' - x_0|)\big)sigmum(x' + x_0).$$

The difference between the values to the right and to the left of the point $x = x_0$ is $-2\beta(1 + \exp(-2\beta x_0))\exp(-2\beta x_0)$, what should be considered a force acting on the cluster. At the same time, however, it is assumed that the charges that make up the cluster are not held by electromagnetic forces that exceed the forces from the electric charges. It can be further shown that the total force acting on a cluster localized near $x = -x_0$, is equal in magnitude and opposite in direction to the force acting on the cluster at the point $x = x_0$ and in the case of symmetric by x the clusters are repelled by the psi functions of the electron, and with an antisymmetric psi function, the clusters are attracted to each other.

In the absence of symmetry of the system, for the forces acting on the clusters, we can obtain: $|F| = 2ab\beta^2\exp(-2\beta x_0)$.

The forces are equal in magnitude and opposite in direction.

We note here that the consideration of this work makes sense when $\beta > 0$, at the same time, the values $\alpha_{1,2}$ not necessarily positive. If $\alpha_2 = -\alpha_1$

then it follows from (6): $\alpha_1 = \frac{\beta}{\sqrt{1-\exp(-4\beta x_0)}}$. Relationship $\frac{a}{b} = \frac{\alpha_1}{\beta-\alpha_1}\exp(-2\beta x_0)$.

4. Systems with spherical symmetry

A stationary spherically symmetric Schrodinger equation with a zero orbital quantum number has the form (a system of units is used in which $m = \hbar = 1$):

$$-\frac{\beta^2}{2}\psi + \frac{1}{2}\Delta\psi = V(r)\psi \qquad (4.1)$$

If the potential created by third-party charges has the form: $V = \frac{\delta(r)}{2r}$, then equation (4.1) is satisfied by the function: $\frac{\exp(-\beta r)}{r}$. In the case of a spherically symmetric Laplace operator $\Delta\frac{e^{-\beta r}}{r} = \frac{\delta(r)}{r^2} + \beta^2 e^{-\beta r}$.

Let's consider the influence of the eigenfield on the state of a system with a potential $\frac{\delta(r)}{2r}$ In equation (4.1), we add the potential determined by the charge: $\Delta V_1 = -\kappa_*|\psi|^2$. Solution for ψ we will search in the form: $\psi = Y(r)\frac{\exp(-\beta r)}{r}$, we denote $Z(r) = rV_1(r)$. We get the system:

$$Y'' - 2\beta Y' = Z, Z'' = -\kappa_*\frac{Y^2}{r}\exp(-2\beta r). \qquad (4.2)$$

The system (4.2) was solved under the conditions: $Y(0) = 0, Z(0) = 0, Z'(0) = -5, Y'(0) = 0$. It was supposed to $\beta = 0.6, \kappa_* = 1$.

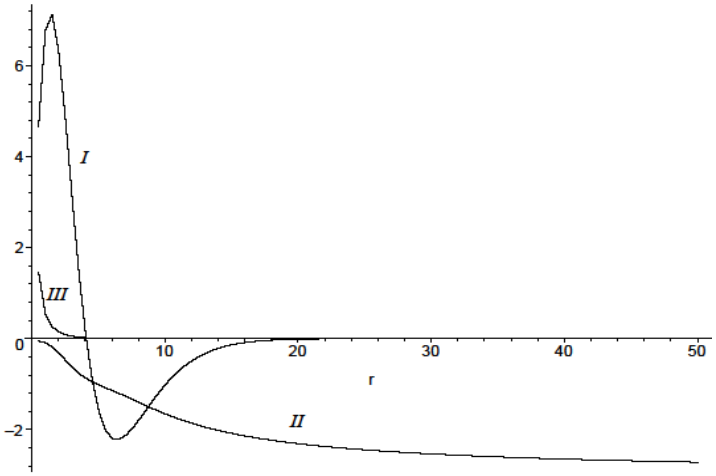

Fig. 3. **Dependence of the potential $\mathbf{V(r)}$ (II), $\boldsymbol{\psi}\mathbf{(r)}$ (I), $\frac{\exp(-\beta r)}{r}$ (III)**

Due to the influence of its own field, the function at the initial site acquires an oscillatory character and decreases exponentially at relatively large r For values of $\beta \leq 0.5$ the system (4.2) detects divergence at the starting point.

The charge density in a spherical cluster is determined from the equation: $4\pi\rho_{cl} = \frac{1}{r^2}\frac{d}{dr}r^2\frac{dV}{dr}$, where from $\rho_{cl} = \frac{1}{r^2}\frac{d}{dr}r^2\frac{dV}{dr}$. Calculating the total charge of the cluster in this case gives an infinite value for the charge. In addition, we note that two separate clusters," locally " spherically symmetric, located at different points, do not affect each other. It should also be noted that the potential defined above does not determine the depth of the associated level, the equation is satisfied for any β. In connection with these circumstances, the setting of the logarithmic derivative is usually used as a boundary condition ψ – is function.

This is an equivalent entry of the Schrodinger equation in the form:

$$i\frac{\partial\Psi}{\partial t} + \Delta\Psi = \frac{2\pi}{\alpha}(\Psi + \vec{r}\nabla\Psi)\delta(\vec{r}), \qquad (4.3)$$

Where is the constant α describes a point potential operator.

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Equation (4.3) describes a system with a single bound state:

$$\Psi = \frac{const}{r}\exp\left(\frac{i\beta^2 t}{2}\right)\exp(-\beta r) = \exp\left(\frac{i\beta^2 t}{2}\right)\psi(r) \quad (4.4)$$

where $\frac{\beta^2}{2}$ - the depth of the only connected level, and it follows from (4.3) that $\beta = \alpha$, that is, the depth of the level is determined by the point interaction operator.

Let there be δ - centers located at $\vec{r} = \pm\vec{r}_0$. The equation for ψ - the function has the form:

$$\frac{1}{2}\Delta\psi = \frac{2\pi}{\alpha}\{[\psi + (\vec{r} - \vec{r}_0)\nabla\psi]\delta(\vec{r} - \vec{r}_0) + [\psi + (\vec{r} + \vec{r}_0)\nabla]\psi\delta(\vec{r} + \vec{r}_0)\}. \quad (4.5)$$

The solution (4.5) can be found in the form:

$$\psi = \frac{\exp(-\beta r_1)}{r_1} + \frac{\exp(-\beta r_2)}{r_2}, \quad (4.6)$$

where $r_{1,2} = |\vec{r} \pm \vec{r}_0|$.

To determine the depth of the level β you can get:

$$\alpha = \beta - \frac{\exp(-2\beta r_0)}{2r_0} \quad (4.7)$$

It can be seen from (4.7) that when $r_0 \to 0$ there is no smooth transition to the bound state of one δ - center. This is how the three-dimensional case differs from the one-dimensional one, where such a transition is possible. Moreover, in the one-dimensional case, the coupling constant is doubled.

Further, let the centers be characterized by different constants - α_1, α_2.

$$i\frac{\partial\Psi}{\partial t} + \frac{1}{2}\Delta\Psi = \frac{2\pi}{\alpha_1}[(\Psi + \vec{r}_1\nabla\Psi)\delta(\vec{r}_1)] + \frac{2\pi}{\alpha_2}[(\Psi + \vec{r}_2\nabla\Psi)\delta(\vec{r}_2)] \quad (4.8)$$

We will look for the solution (4.8) in the form: $\Psi = \exp\left(\frac{i\beta^2 t}{2}\right)\psi(\vec{r})$,

$$\psi = \frac{a}{r_1}\exp(-\beta r_1) + \frac{b}{r_2}\exp(-\beta r_2) \quad (4.9)$$

Substituting this expression in (4.8) and equating the coefficients for $\delta(r_1), \delta(r_2)$, we get the relations:

$$-a = -\frac{\beta a}{\alpha_1} + \frac{b}{2\alpha_1 r_0}\exp(-2\beta r_0), -b = -\frac{\beta b}{\alpha_2} + \frac{a}{2\alpha_2 r_0}\exp(-2\beta r_0). \quad (4.10)$$

The condition for the presence of a non-zero solution of this system has the form:

$$(\beta - \alpha_1)(\beta - \alpha_2) = \frac{\exp(-4\beta r_0)}{4r_0^2}. \quad (4.11)$$

Note that when $\alpha_1 = \alpha_2 = \alpha$ easy to get $\alpha = \beta \pm \frac{\exp(-4\beta r_0)}{4r_0^2}$.

5. Integral model of point interaction

Let us consider such a representation of the point interaction, which instead of the derivative contains an integral of ψ – is function. Let the Schrodinger equation have the form:

$$i\frac{\partial\Psi}{\partial t} + \frac{1}{2}\Delta\Psi = -\frac{1}{2}\int K(\vec{r},\vec{r}')dr'\Psi(\vec{r}',t)D(\vec{r}), \quad (5.1)$$

where $K(\vec{r},\vec{r}') = \frac{1}{|\vec{r}-\vec{r}'|}$. In the left part (5.1) there are three types of features: $\frac{\delta(r)}{r^2}, \frac{\delta(r)}{r}, \delta(r)$. According to this function $D(r)$ let's imagine it in the form: $D = \alpha\frac{\delta(r)}{r^2} + \alpha_1\frac{\delta(r)}{r} + \alpha_2\delta(r)$. Three equations follow from (5.1):

$$1 = \frac{\alpha}{\beta}, -\beta = \frac{\alpha_1}{\beta}, \frac{\beta^2}{2} = \frac{\alpha_2}{\beta} - \frac{\alpha_1}{2} + \frac{\alpha\beta}{6}. \quad (5.2)$$

As a result, we get:

$$D = \alpha\frac{\delta(r)}{r^2}\left(1 - \frac{\alpha r}{2} + \frac{(\alpha r)^2}{12}\right). \quad (5.1)$$

has a solution that coincides with (4.6), and the depth of the level β defined by a constant α.

Consider two point centers localized at points and characterized by a constant α In this case, the Schrodinger equation has the form

$$i\frac{\partial\Psi}{\partial t} + \frac{1}{2}\Delta\Psi = -\frac{1}{2}\int \frac{d\vec{r}'\Psi(\vec{r}',t)}{|\vec{r}-\vec{r}'|}D(\vec{r}_1,\vec{r}_2) \quad (5.3)$$

where $\vec{r}_{1,2} = \vec{r} \pm \vec{r}_0$, function D we define it as follows: $D = \alpha\left(\frac{\delta(r_1)}{r_1^2} + \frac{\delta(r_2)}{r_2^2}\right) + \gamma_1\left(\frac{\delta(r_1)}{r_1} + \frac{\delta(r_2)}{r_2}\right) + \gamma_2(\delta(r_1) + \delta(r_2))$. When substituting the expression (4.6) into (4.14), taking into account the ratio: $\frac{d\vec{r}'}{|\vec{r}-\vec{r}'|r'_{1,2}}\exp\int(-\beta r'_{1,2}) = \frac{4\pi}{\beta^2 r_{1,2}}[1 - \exp(-\beta r_{1,2})]$, (where $\vec{r}'_{1,2} = \vec{r}' \pm \vec{r}_0, \vec{r}_{1,2} = \vec{r} \pm \vec{r}_0$) equating the coefficients for $\delta(r_1)$ we get the ratio: $\frac{\delta(r_1)}{r_1^2}\left[1 - \beta r_1 + \frac{(\beta r_1)^2}{2}\right] = \frac{\delta(r_1)}{r_1^2}(\alpha + \gamma_1 r_1 + \gamma_2 r_1^2)\left[\frac{1}{\beta}\left(1 - \frac{\beta r_1}{2} + \frac{\beta^2 r_1^2}{6}\right) + \frac{1}{2\beta r_0}\left(1 - e^{2\beta r_0}\right)\right]$.

Hence the equalities follow:

$$1 = \frac{\alpha}{\beta}\left(1 + \frac{1}{2\beta r_0}(1 - \exp(-2\beta r_0))\right), \quad (5.4)$$

$$-\beta = \frac{\gamma_1}{\beta} - \frac{\alpha}{2}, \frac{\beta^2}{2} = -\frac{\gamma_2}{\beta} - \alpha\beta. \quad (5.5)$$

From (5.4) follows the relation connecting the constant of the point interaction with the depth of the level:

$$\alpha = \frac{\beta}{1 + \frac{1}{2\beta r_{00}}[1 - \exp(-2\beta r_0)]} \quad (5.6)$$

The equalities (5.5) defining the structure of the two-point operator do not affect the relation (5.6). It follows from (5.6) that when $r_0 \to 0$ $\alpha = 2\beta$, that is, the connected state continuously passes into a state with a doubled connection constant. This representation of the point interaction adequately describes the physical situation under consideration.

Let us further consider the case when point interactions are described by different coupling constants – α_1, α_2. Let's define the associated state of such a system. The Schrodinger equation has the form:

$$i\frac{\partial\Psi}{\partial t} + \frac{1}{2}\Delta\Psi = -\frac{1}{2}\int \frac{d\vec{r}'\Psi(\vec{r}',t)}{|\vec{r}-\vec{r}'|}[\alpha_1\delta(\vec{r} - \vec{r}_0) + \alpha_2\delta(\vec{r} + \vec{r}_0)]. \quad (5.7)$$

We will look for a solution in the following form:

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$$\Psi = \exp\left(\frac{i\beta_1^2 t}{2}\right)\left[\frac{a_1\exp(-\beta_1|\vec{r}-\vec{r}_0|)}{|\vec{r}-\vec{r}_0|} + \frac{b_1\exp(-\beta_1|\vec{r}+\vec{r}_0|)}{|\vec{r}+\vec{r}_0|}\right] + \exp\left(\frac{i\beta_2^2 t}{2}\right)\left[\frac{a_2\exp(-\beta_2|\vec{r}-\vec{r}_0|)}{|\vec{r}-\vec{r}_0|} + \frac{b_2\exp(-\beta_2|\vec{r}+\vec{r}_0|)}{|\vec{r}+\vec{r}_0|}\right] \quad (5.8)$$

Substituting (5.8) into (5.1) leads to a system of equations:

$$a_{1,2} = a_{1,2}\frac{\alpha_1}{\beta_{1,2}} + b_{1,2}\frac{\alpha_2}{2\beta_{1,2}^2 r_0}\left[1 - \exp(-2\beta_{1,2}r_0)\right], \quad (5.9)$$

$$b_{1,2} = a_{1,2}\frac{\alpha_1}{2\beta_{1,2}^2 r_0}\left[1 - \exp(-2\beta_{1,2}r_0)\right] + b_{1,2}\frac{\alpha_2}{\beta_{1,2}}.$$

Conditions for the existence of nonzero solutions for pairs of quantities a_1, b_1 and a_2, b_2 they have the form:

$$\left(1 - \frac{\alpha_1}{\beta_1}\right)\left(1 - \frac{\alpha_2}{\beta_1}\right) = \frac{\alpha_1\alpha_2}{4\beta_1^4 r_0^2}[1 - \exp(-2\beta_1 r_0)]^2, \quad (5.10)$$

$$\left(1 - \frac{\alpha_1}{\beta_2}\right)\left(1 - \frac{\alpha_2}{\beta_2}\right) = \frac{\alpha_1\alpha_2}{4\beta_2^4 r_0^2}[1 - \exp(-2\beta_2 r_0)]^2.$$

If $r_0 \to 0$, that $\beta_1 = \beta_2 = \alpha_1 + \alpha_2$. For two closely lying centers, there is only one connected level. The possibility of the existence of two levels of different depths is determined by the difference in the solutions of the equation for β:

$$\left(1 - \frac{\alpha_1}{\beta}\right)\left(1 - \frac{\alpha_2}{\beta}\right) = \frac{\alpha_1\alpha_2}{4\beta^4}[1 - \exp(-2\beta r_0)]. \quad (5.11)$$

For large values r_0 there are 2 solutions: $\beta = \alpha_1$ and $\beta = \alpha_2$.

Conclusion

In this paper, one-dimensional and spherically symmetric three-dimensional systems with point interaction are studied. In one-dimensional systems, the problem of accounting for the eigenfield and the model non-stationary problem are solved. The concept of a point cluster is introduced and the interaction of such clusters is considered. In 3-dimensional systems, an integral model of point interaction is proposed that adequately describes the merger of two centers.

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